Operational Risk Modeling

RMA Training (part 2) March 2013

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About the Speaker

- Senior Expert – McKinsey & Co
- Implemented Operational Risk Quantification Methodology for Multiple Clients
- 10 years of risk quantification experience in Credit and Operational Risk
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Scenario Analysis: Goals

- Forward looking assessment of risk
- Reduce risks by
  - Risk avoidance
  - Hedging/Insurance
  - Improvement of controls
- Cross validate data
- Supplement data by modeling rare events
- Stress test
- **Make decisions on the maximum loss**
- Explore correlation between different business lines

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Scenario Analysis: Implementation

- Typically derived by panel of experts
- Several Operational Risk Loss databases can assist in scenario workshops
- Potential approach to quantification
  - Create a set of loss generating scenarios
  - Split loss amounts into buckets, e.g. losses form $0 to 10K, from 10K to 50K, ets
  - Assign frequencies to each bucket, i.e. "How many losses less than 100K bank will have in next year" or "Once in how many years bank may have a loss exceeding 1B"

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Scenario Analysis: Implementation

The scenario table can be used to do a simulation.

Bucket	Scenario Frequency	Theoretical Frequency
0-10,000	100	100.90
10,001-100,000	12	12.94
100,001-1,000,000	2	2.21
1,000,001-10,000,000	0.1	0.18
10,000,001-100,000,000	0.01	0.01
>100,000,000	0	0.00
Total Frequency	**114.11**	**116.24**

- **Frequency** is the sum of number of losses in each bucket
- **Severity is either**
  1. Use scenario table directly as an empirical severity function
  2. Smooth the buckets by fitting theoretical distribution

The whole set of severity distributions can be tried. Distribution most closely approximating the scenario is chosen.

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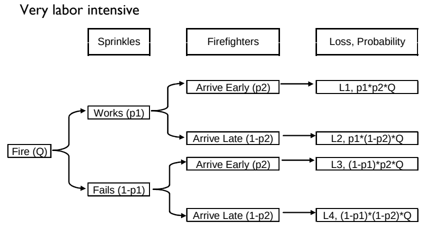
Scenario Analysis: Different Approach

Event Tree Analysis

- Is very useful for understanding the process
- May be used to create artificial data
- Both, frequency and severity distributions can be constructed
- Very labor intensive



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Combining Data and Scenario

1. Artificial data
   i. Collect empirical data
   ii. Create scenarios
   iii. Verify that scenario and data are consistent
   iv. Use scenario data to create artificial data points
   v. Combine artificial and real data
   vi. Model frequency and severity
   vii. Use Monte Carlo simulation to get aggregate loss distribution

The problem there is a lot of flexibility in creation of artificial data points from scenarios.
Unfortunately the way the data is created can significantly influence capital number.

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Combining Data and Scenario

2. Separate tail and body distributions
   i. Collect empirical data
   ii. Create scenarios
   iii. Verify that scenario and data are consistent
   iv. Use data to create truncated body distribution. Use some threshold as the maximum of distribution. May be the largest realized loss as the maximum of distribution
   v. Use scenario table and/or data above the threshold to fit a tail distribution. Utilize truncated distribution with minimum equal the maximum of body distribution.
   vi. Determine appropriate frequencies of body and tail distributions
   vii. Simulate as two independent business lines and add losses

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Case Study

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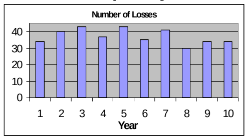
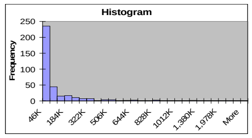
Data

Frequency



- 371 losses are recorded
- 10 year time horizon
- Number of losses does not vary significantly

Severity



- Losses less than $1,000 are not recorded
- The maximum realized loss is 4,678,884

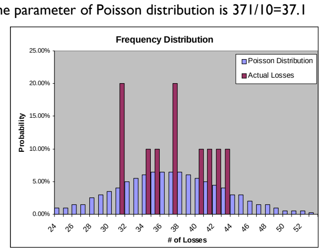
Scenario

Scenario Table

Bucket	Scenario Frequency	Actual Frequency
1,000--10,000	10	10.8
10001--100,000	20	17.6
100,001--1,000,000	5	7.7
1,000,001--5,000,000	1	1
5,000,001--10,000,000	1	0
10,000,001--50,000,000	0.01	0
50,000,001--100,000,000	0.001	0
>100,000,000	0	0
Total Frequency	37.011	37.1

Frequency

- The frequency distribution is stable over the time. The C.V of number of losses is small, 0.11.
- Poisson distribution can be used.
- The parameter of Poisson distribution is 371/10=37.1



Severity: Data

- Option 1: Shift data by subtracting $1,000 from data points

Lognormal		Transformed Beta	
mu	10.10	alpha	8.16
sigma	1.93	beta	9.56
		lambda	11.29
		c	0.26
mean	158,195	mean	174,870
st dev	1,000,116	st dev	4,999,910
KS	2.90%	KS	2.50%
KS@20%	5.56%	KS@20%	5.56%

- Option 2: Fit truncated distribution with min $1,000

Lognormal		Transformed Beta	
mu	10.04	alpha	3.25
sigma	1.91	beta	16.82
		lambda	6.57
		c	0.36
mean	142,802	mean	413,007
st dev	875,086	st dev	infinity
KS	2.80%	KS	2.80%
KS@20%	5.56%	KS@20%	5.56%

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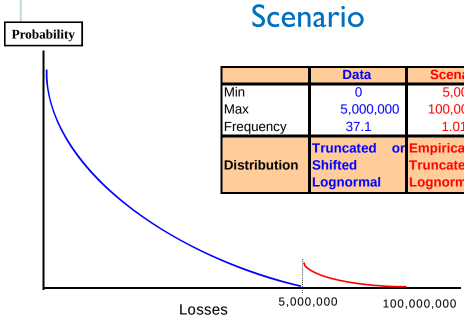
Severity: Combining Data & Scenario



	Data	Scenario
Min	0	5,000,001
Max	5,000,000	100,000,000
Frequency	37.1	1.011
Distribution	**Truncated or Shifted Lognormal**	**Empirical or Truncated Lognormal**

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Capital

Percentile	Body/Data	Tail/Scenario	Combined
Average	4,647,300	9,482,200	14,129,000
50%	4,211,000	6,922,000	11,659,000
97%	9,826,000	34,820,000	39,731,000
99%	11,577,000	44,573,000	49,628,000
99.50%	12,601,000	50,845,000	55,903,000
99.90%	14,896,000	64,730,000	69,892,000
99.95%	15,873,000	71,926,000	77,091,000
99.97%	16,661,000	77,670,000	81,982,000
99.99%	17,779,000	88,214,000	93,156,000

- The capital number is dominated by the tail of the distribution
- The dominance of the tail increases, with the increase of risk level
- In any distribution with no maximum the influence of tail will be larger
- The average of the data portion matches the average of realized losses, but few large tail losses will shift the Expected loss

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Dependency

Background

- Most of the banks have more than one Business line or event type
- Standard requirement is to add capital number of different lines , which may be punitive and result higher charges than in Standard Measurement Approach
- However correlation modeling is allowed, provided sound approach
- Addition of capital numbers implies perfect correlation
- The perfect correlation assumption is implausible in most of the cases

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Measures Of Dependency

- Pearson
  - Classical linear correlation, *if x increases by 1 unit, by how many units y will change*
  - Based on existence of second moment, and may not exist for fat tailed distributions
- Spearman
  - Linear correlation of the ranks, *if x takes its third largest value, what value y will take*
  - Always exists
- Kendall-Tau
  - Measure of concordance, *if x increases (decreases) will y increase (decrease)*
  - Alternative to Spearman

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Techniques To Model Dependency

- Complete Multivariate Distribution Function Specification
  - Fully describes the stochastic process
  - Generally not feasible, since essentially perfect data is required
  - Analytics may not exist for types of distributions used in operational risk measurement
- Empirical Dependency Structure
  - Implemented as a copula
  - Requires less data
  - Lends itself to scenario analysis
  - Emphasis on tail dependency, i.e. Low probability events

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Definition And Types Of Copulas

A function that joins univariate distribution functions to form multivariate distribution functions. A copula C(x1, … , xn) is defined as the multivariate distribution function of a random vector with uniform-[0,1] marginals.

- Types of Copulas:
  - Elliptical
    - Gaussian
    - Student-t
  - Archimedean
    - Gumbel
    - Joe-Clayton
  - Many others…

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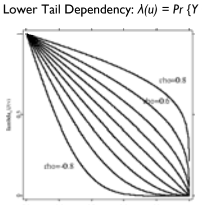
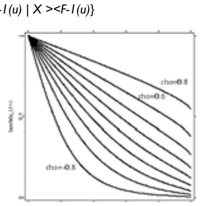
Tail Dependency And Risk Management

- Risk Management Focused On Tail Events
- Tail Dependency
- X, Y Sources Of Risk With Distributions F(X), G(Y)
- Upper Tail Dependency: $\lambda(u) = Pr\ \{Y > G\text{-}1(u) \mid X > F\text{-}1(u)\}$
- Lower Tail Dependency: $\lambda(u) = Pr\ \{Y < G\text{-}1(u) \mid X >< F\text{-}1(u)\}$



Bivariate Normal Copula



Bivariate t-Copula

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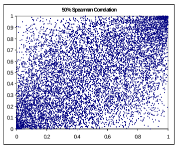
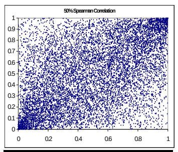
Tail Dependency of Various Copulas

Normal Copula



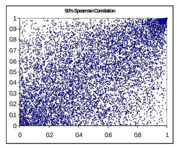
Level	Tail Dependency
90%	32.57%
95%	25.61%
99%	16.59%
99.5%	12.27%
99.9%	11.76%

t Copula



Level	Tail Dependency
90%	37.50%
95%	32.06%
99%	23.48%
99.5%	24.70%
99.9%	18.97%

Gumbel Copula



Level	Tail Dependency
90%	46.29%
95%	45.00%
99%	44.40%
99.5%	43.98%
99.9%	50.00%

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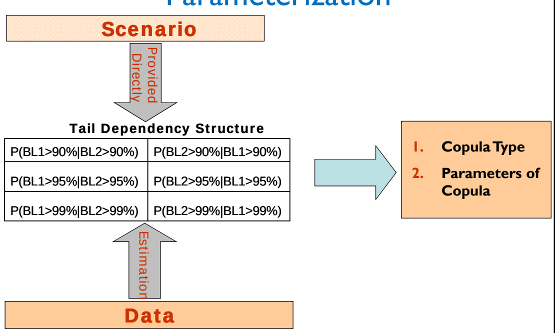
Parameterization



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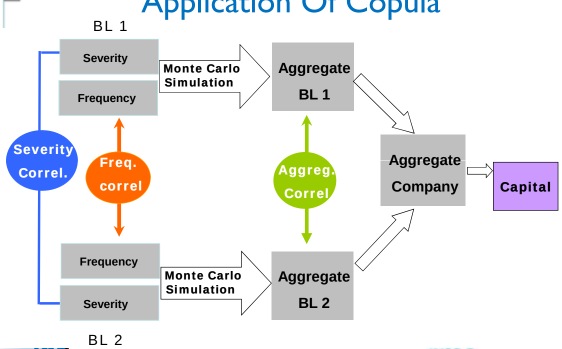
Application Of Copula



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What to "Correlate" Frequency Or Severity ?

- Frequency?
  - Impossible to calibrate empirically
  - Intuitively appealing
- Severity?
  - May be possible to calibrate from data, but
  - The exact date of the loss is not defined
  - Different dating on the losses results different severity dependence estimates
  - Difficulty correlating severities in business lines with different frequencies
- Aggregate?
  - Follows Basel II aggregation logic
  - Requires less parameters
  - Makes the effect of dependency explicit

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Thank You

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